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Optimal control of the heat storage in a porous slab

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INTRODUCTION

Investigation of the nonthermal equilibrium flow of a fluid through a packed bed is of interest because of the important applications of porous beds. These applications include the storage of heat energy, chemical reactors, and adsorption and absorption operations [1]. In recent works [1–4] a general set of volume-averaged governing equations for nonthermal equilibrium, forced fluid flow through sensible and latent heat storage beds is presented and comprehensive numerical analyses of the phenomena are carried out. In these references the two-energy-equation model is utilized in which the temperature difference between the fluid and solid phases is taken into account. Proceeding from this model, ref. [5] discusses some energy characteristics of the thermal charging and discharging of packed beds. In ref. [6], the temperature difference between the fluid and solid phases is analyzed and found to exhibit wave properties.

Most of the analytical studies of these phenomena are concentrated on the Schumann model of a packed bed, suggested in ref. [7]. In the Schumann model a flow of incompressible fluid through a packed bed is considered and the thermal conduction terms in both the fluid and solid phase energy equations are neglected. In the present paper we follow this model. The following assumptions are employed:

- the fluid phase is incompressible and the mass flow rate at every cross-section of the packed bed is constant;
- thermal, physical and transport properties are independent of temperature and location;
- conductive heat transfer is negligible within both the fluid and solid phases;
- heat transfer and fluid flow are one-dimensional;
- the fluid phase is initially in thermal equilibrium with the solid phase.

As follows from ref. [8], under these assumptions the equations governing the solid and fluid temperature distributions can be presented in the following nondimensional form:

$$\frac{\partial \theta}{\partial t} = \phi - \theta \tag{1a}$$

$$\Lambda \frac{\partial \phi}{\partial t} + \frac{\partial \phi}{\partial z} = \theta - \phi \tag{1b}$$

where

$$\Lambda = \frac{\varepsilon \rho_f c_{pf}}{(1-\varepsilon)\rho_s c_s}.$$

Here the dimensionless temperature of the solid phase is defined as

$$\theta(z,t) = \frac{T_s - T_1}{T_2 - T_1} \tag{2a}$$

and the dimensionless temperature of the fluid phase is defined as

$$\phi(z,t) = \frac{T_f - T_1}{T_2 - T_1} \tag{2b}$$

where T_1 and T_2 are reference temperatures chosen to suitably normalize the initial and boundary conditions. If $T_s(0,0) \neq T_f(0,0)$, T_1 and T_2 can be selected as

$$T_1 = T_s(0,0) \tag{3a}$$

and

$$T_2 = T_f(0,0). \tag{3b}$$

The dimensionless time and coordinate in equations (1a) and (1b) are defined as follows:

$$t = \frac{h_{sf} a_{sf} t'}{(1-\varepsilon)\rho_s c_s}$$

and

$$z = \frac{h_{sf} a_{sf} z'}{\varepsilon \rho_f c_{pf} v}.$$

The analytical solutions for equations (1a) and (1b) for different boundary conditions are obtained in refs. [9–13]. In ref. [14] the solution for the case when the entrance fluid temperature is a function of time is obtained. In this reference, the following initial and boundary conditions are utilized:

$$\theta(z,0) = \theta_0(0) \tag{4a}$$

$$\phi(0,t) = \phi_{in}(t). \tag{4b}$$

Upon simple rearrangement, the solution obtained in ref. [14] can be put into the following form:

$$\theta(z,t) = \exp(-z)\int_0^{t-\Lambda z} \phi_{in}(t-\Lambda z-\tau)$$

$$\times \exp(-\tau) I_0\left[(4\tau z)^{1/2}\right] d\tau$$

$$+\exp(\Lambda z - t)\left[\theta_0(z) + \int_0^z \theta_0(z-\xi)\right.$$

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NOMENCLATURE

a_{sf}	specific surface area common to solid and fluid phases [m^2/m^3]
c_p	specific heat at constant pressure [J kg^{-1} K^{-1}]
h_{sf}	fluid-to-solid phase heat transfer coefficient [W m^{-2} K^{-1}]
I_v	modified Bessel function of the order v
L	dimensionless length of the porous slab
L'	length of the porous slab [m]
t	dimensionless time
t'	time [s]
t_f	dimensionless duration of heating
T	temperature [K]
u_{min}	the lower boundary for admissible controls
u_{max}	the upper boundary for admissible controls
v	velocity of the fluid phase [m s^{-1}]
z	dimensionless Cartesian coordinate
z'	Cartesian coordinate [m].

Greek symbols

ε	porosity
ϕ	dimensionless temperature of the fluid phase
ϕ_{in}	dimensionless inlet temperature of the fluid phase
$\hat{\phi}_{in}$	optimal inlet temperature of the fluid phase
$\Phi(\phi_{in})$	the performance functional
λ_1	the Lagrange multiplier
Λ	the ratio of the effective heat capacities for the fluid and solid phases
θ	dimensionless temperature of the solid phase
θ_0	dimensionless initial temperature of the solid phase
ρ	density [kg m^{-3}].

Subscripts

f	fluid
s	solid.

$$\times \exp(-\xi)\left(\frac{t-\Lambda z}{\xi}\right)^{1/2} I_1\left[\{4\xi(t-\Lambda z)\}^{1/2}\right]\mathrm{d}\xi\Bigg] \quad (5)$$

$$\phi(z,t) = \exp(\Lambda z - t)\int_0^z \theta_0(z-\xi)$$

$$\times \exp(-\xi) I_0\left[\{4\xi(t-\Lambda z)\}^{1/2}\right]\mathrm{d}\xi$$

$$+\exp(-z)\left[\phi_{in}(t-\Lambda z)+\int_0^{t-\Lambda z}\phi_{in}(t-\Lambda z-\tau)\right.$$

$$\left.\times \exp(-\tau)\left(\frac{z}{\tau}\right)^{1/2} I_1[(4\tau z)^{1/2}]\,\mathrm{d}\tau\right]. \quad (6)$$

Equations (5) and (6) determine the temperatures of the solid and fluid phases at a particular point in the porous bed with the position z' (or corresponding dimensionless coordinate z), after this point is reached by the temperature front moving from the fluid inlet boundary with a velocity v, i.e. when $t' \geqslant z'/v$. In the dimensionless coordinates this condition is $t \geqslant \Lambda z$. Because the thermal conductivity within both the solid and fluid phases is neglected, for $t < \Lambda z$ the temperature of the solid phase at this point equals the initial temperature determined by the function θ_0 (z) in equation (4a).

STATEMENT AND SOLUTION OF THE OPTIMAL-CONTROL PROBLEM

Consider a one-dimensional porous slab of the length L'. The dimensionless length of the slab is then defined as

$$L = \frac{h_{sf}a_{sf}L'}{\varepsilon\rho_f c_{pf} v}.$$

It is assumed that the initial temperature of the slab is uniform. If the reference temperatures, T_1 and T_2, are chosen according to equations (3a) and (3b), then according to equation (2a) the dimensionless initial temperature of the solid phase equals zero. This essentially simplifies equations (5) and (6), because in this case the second term on the right-hand side of equation (5) and the first term on the right-hand side of equation (6) equal zero.

It is assumed that the dimensionless fluid temperature at the entrance to the porous slab is given by some function of time, $\phi_{in}(t)$. Since there is a temperature difference between the fluid and solid phases, the outlet fluid temperature is higher than the temperature of the solid phase at the fluid outlet. In other words, a part of the heat that could be stored in the porous slab leaves the slab with the fluid flow. To increase the efficiency of heat storage in the packed bed it is important to find a way to minimize this loss of the heat energy.

Consider the following optimization problem. As the optimization criterion, the amount of heat energy stored in the slab is used. It is necessary to maximize the amount of heat stored in the porous slab under the following constraints: (a) a given amount of heat can be supplied by the incoming fluid flow and (b) there is a given duration of the process. As the control the inlet fluid flow temperature, $\phi_{in}(t)$, is considered. It is assumed that this function is a bounded, piecewise continuous function with a minimum value u_{min} and a maximum value u_{max}. The minimum value corresponds to the fluid temperature in the 'cold tap' and the maximum value corresponds to the fluid temperature in the 'hot tap'.

The mathematical formulation of this problem is as follows. It is necessary to determine the optimal temperature $\hat{\phi}_{in}(t)$ that maximizes the following performance functional:

$$\Phi(\phi_{in}) = \int_0^L \theta(z,t_f)\,\mathrm{d}z \rightarrow \max \quad (7)$$

where the function $\theta(z,t_f)$ is determined by equation (5), under the following constraints

$$\int_0^{t_f}\phi_{in}(\tau)\,\mathrm{d}\tau = E = \text{const} \quad (8)$$

and

$$u_{min} \leqslant \phi_{in}(t) \leqslant u_{max}. \quad (9)$$

To bring the problem (7)–(9) to the form of an optimal-control problem it is necessary to rearrange the functional (7). To accomplish this, equation (5) for the function $\theta(z,t)$ is first rearranged by the following change of the integration variable:

$$\tau^* = t_f - \tau - \Lambda z. \quad (10)$$

Then, accounting for the assumption that the initial temperature of the slab is uniform, equation (5) at the moment $t = t_f$ can be written as

$$\theta(z, t_f) = \exp(-z)\int_0^{t_f-\Lambda z} \phi_{in}(\tau^*)$$

$$\times \exp(-t_f+\tau^*+\Lambda z)I_0[\{4(t_f-\tau^*-\Lambda z)z\}^{1/2}]\,d\tau^*. \quad (11)$$

For $t_f < \Lambda L$ the temperature front has not yet reached the outlet boundary and no heat has yet been lost with the fluid leaving the slab. Therefore we consider only the case $t_f \geqslant \Lambda L$.

Consider the function

$$\Psi(z,\tau) = \begin{cases} \exp(-z-t_f+\tau+\Lambda z)I_0[\{4(t_f-\tau-\Lambda z)z\}^{1/2}] \\ \quad \text{if } 0 \leqslant \tau \leqslant t_f-\Lambda z \\ 0 \\ \quad \text{if } \tau > t_f-\Lambda z. \end{cases} \quad (12)$$

Then, utilizing equations (11) and (12) and changing the integration order, equation (7) can be recast as

$$\Phi(\phi_{in}) = \int_0^L \theta(z, t_f)\,dz = \int_0^{t_f} \phi_{in}(\tau)\Xi(\tau)\,d\tau \rightarrow \max \quad (13)$$

where

$$\Xi(\tau) = \int_0^L \Psi(z,\tau)\,dz.$$

The problem given by equations (8), (9) and (13) is an optimal-control problem. It can be solved by the minimum principle of Pontryagin considered, for example, in refs. [15, 16].

Application of this principle leads to the following requirement:

$$\hat{\phi}_{in}(t)[\lambda_1-\Xi(t)] \rightarrow \min \quad (14)$$

where λ_1 is the Lagrange multiplier.

Equation (14), when applied accounting for the constraint (9), makes it possible to determine the optimal temperature, $\hat{\phi}_{in}(t)$, as

$$\hat{\phi}_{in}(t) = u_{min} \quad \text{if } \lambda_1-\Xi(t) > 0$$
$$\hat{\phi}_{in}(t) = u_{max} \quad \text{if } \lambda_1-\Xi(t) < 0. \quad (15)$$

To make use of equations (15) it is necessary to calculate the value of the Lagrange multiplier, λ_1. To do this, the transcendental equation (8) needs to be solved accounting for equations (15). To solve this problem, first a segment that unequivocally contains the desired value of λ_1 was selected. Then an algorithm for finding a root of a transcendental equation on a given segment was applied to equation (8).

Figure 1 depicts the optimal controls, $\hat{\phi}_{in}(t)$, for different durations of heating, t_f, for the following data: $u_{min} = 0$, $u_{max} = 2$, $E = t_f$, $L = 1$, $\Lambda = 0.05$. As it can be seen in Fig. 1, for a small duration of heating ($t_f = 0.08$) the optimal fluid inlet temperature $\hat{\phi}_{in}(t)$ first takes its maximum value u_{max} and then its minimum value u_{min}. With an increase in the duration of heating ($t_f = 0.2$) a qualitative change in the behavior of the optimal inlet temperature takes place. Now, the optimal inlet temperature first takes its minimum value, then the maximum value, and then again the minimum value. With a further increase in the heating duration ($t_f = 2$) this qualitative behavior remains, but the duration of the third segment decreases.

Thus, Fig. 1 shows that with an increase in the duration of heating the transition from the first type of behavior of the optimal inlet temperature (maxima–minima) to the second type (minima–maxima–minima) takes place. To understand qualitatively the reason for this transition we consider two extreme cases, namely, a very short and a very long duration of heating. For a very short duration of heating the temperature front has just reached the outlet boundary of the porous slab and the maxima–minima behavior is obviously beneficial. This is because if the hot fluid is supplied in the beginning of the process, the time of contact of the hot fluid and the solid phase is the longest. Consequently, more heat energy can be transferred to the solid phase.

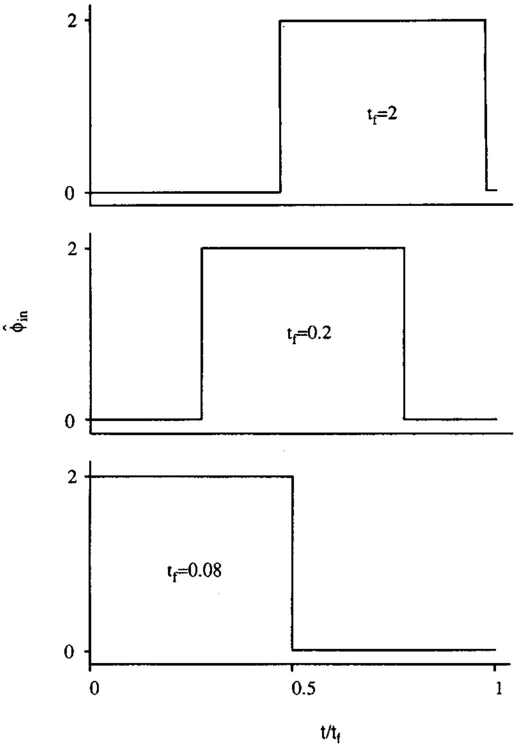

Fig. 1. The optimal fluid inlet temperature for different durations of heating as a function of time.

Contrarily, for a very long duration of heating the hot fluid should be supplied at the end of the process. Then the final temperature of the solid phase will be approximately uniform and nearly equal the temperature of the hot fluid.

For a mean duration of heating the hot fluid should be supplied sometime between the beginning and the end of the process. This leads to the minima–maxima–minima behavior.

It is interesting to compare the value that the performance functional $\Phi(\phi_{in})$ takes on the optimal functions shown in Fig. 1 and on the unit functions $\phi_{in}^*(t) \equiv 1$. These unit functions correspond to a constant inlet temperature of the fluid phase. It is easy to show that for $E = t_f$ (this was used to calculate Fig. 1) the functions $\phi_{in}^*(t) \equiv 1$ also satisfy constraint (8). Calculations show the following: for $t_f = 0.08$, $\phi(\hat{\phi}_{in})/\Phi(\phi_{in}^*) = 1.320$ (the gain in the amount of the heat energy stored when the optimal inlet temperature is applied instead of the constant inlet temperature is 32.0%), for $t_f = 0.2$, $\Phi(\hat{\phi}_{in})/\Phi(\phi_{in}^*) = 1.119$ (the gain is 11.9%) and for $t_f = 2$, $\Phi(\hat{\phi}_{in})/\Phi(\phi_{in}^*) = 1.305$ (the gain is 30.5%). Thus, utilizing the optimal fluid inlet temperature makes it possible to increase the amount of heat energy stored in the porous slab. We underline again that this is reached due to the decrease of the amount of heat which is thrown away when the fluid flow leaves the slab.

CONCLUSIONS

(a) A method for the optimization of heating a one-dimensional porous slab is suggested. This method makes it possible to optimize the performance of the packed bed with respect to the amount of heat stored in it.

(b) It is shown that with an increase in the duration of heating a qualitative change in the behavior of the optimal inlet temperature takes place. For practical applications it means that depending on the duration of the process the inlet temperature should be controlled in two different ways. For a small duration of heating, the optimal fluid inlet temperature first takes its maximum value and then its minimum value. In contrast to this, for the longer duration of heating the optimal inlet temperature first takes its minimum value, then the maximum value, and then again the minimum value. With a further increase in the duration of the heating this qualitative behavior remains, but duration of the third segment decreases.

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